Abstractions of Dynamical Systems

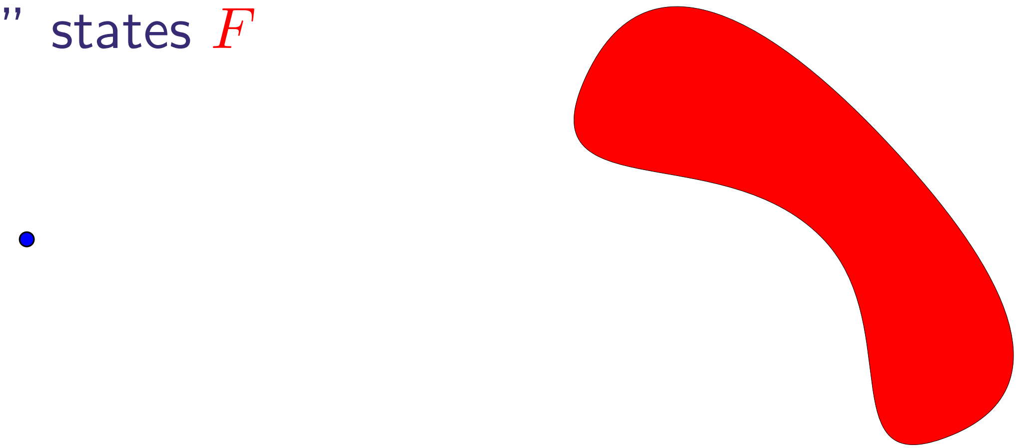
Colas Le Guernic

October 28, 2010

Motivations

A typical example:

- a differential equation $\dot{x} = f(x)$, $f : \mathbb{R}^d \longrightarrow \mathbb{R}^d$
- an initial point x_0
- a set of "bad" states F



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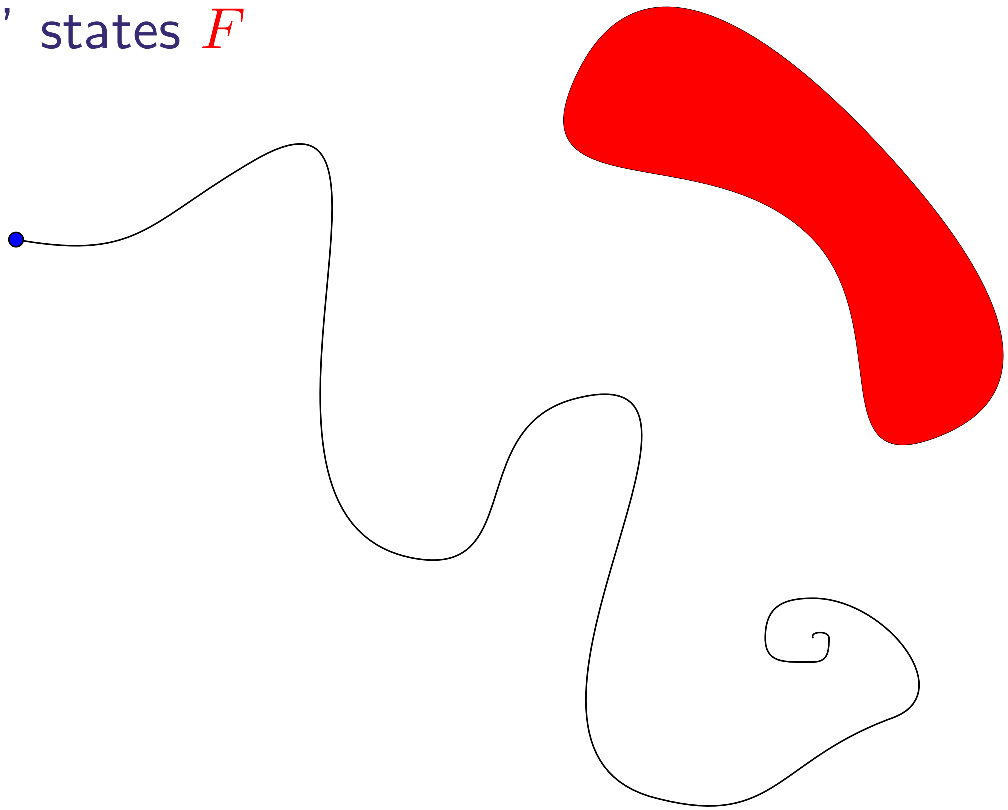

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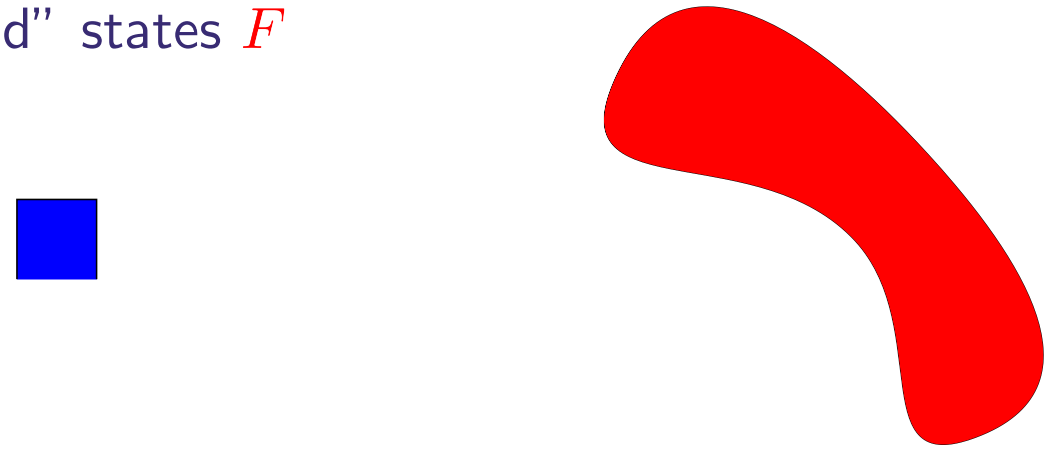

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A typical example:

- a differential inclusion $\dot{x} \in f(x)$, $f : \mathbb{R}^d \to \mathcal{P}\left(\mathbb{R}^d\right)$
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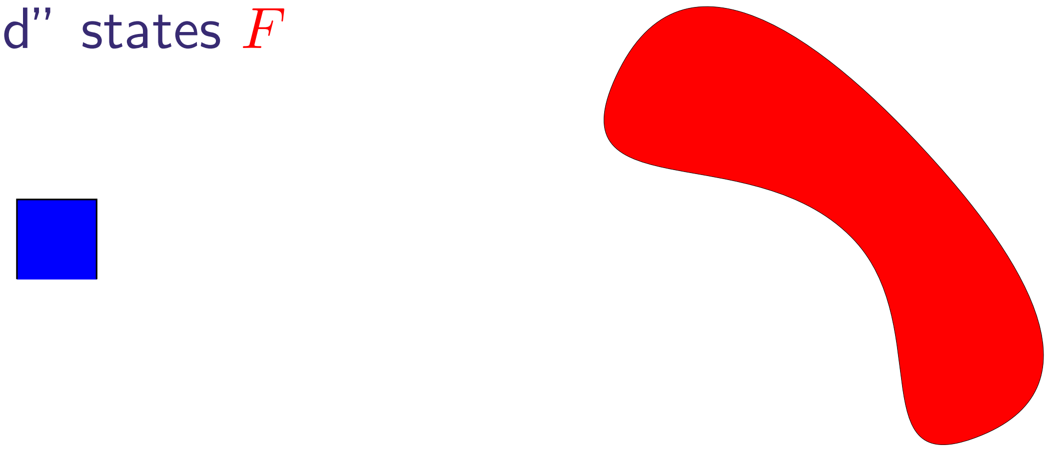

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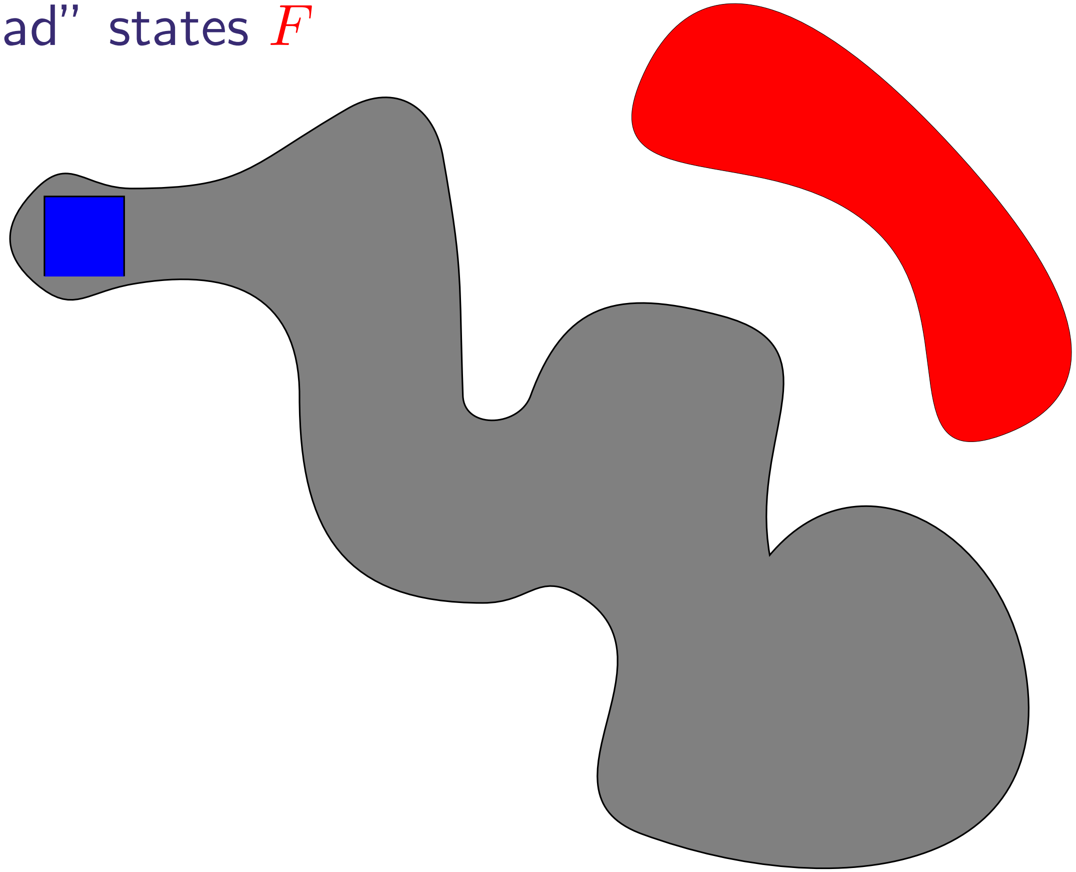

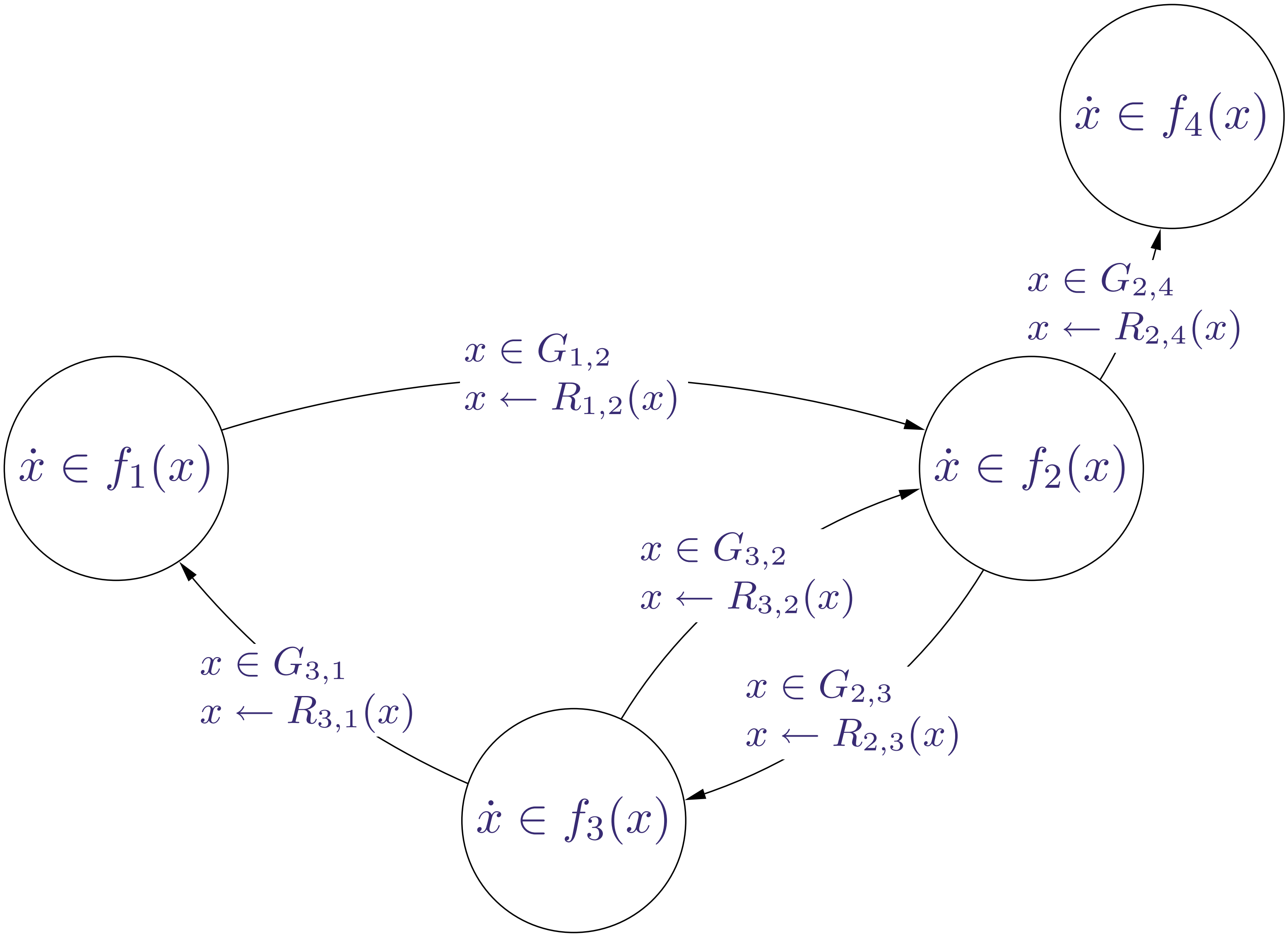
$\dot{x} \in f_4(x)$
$x \in G_{2,4}$
$x \leftarrow R_{2,4}(x)$
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$x \leftarrow R_{3,1}(x)$
$x \in G_{2,3}$
$x \leftarrow R_{2,3}(x)$
$\dot{x} \in f_3(x)$

Outline

A few reflexions on:

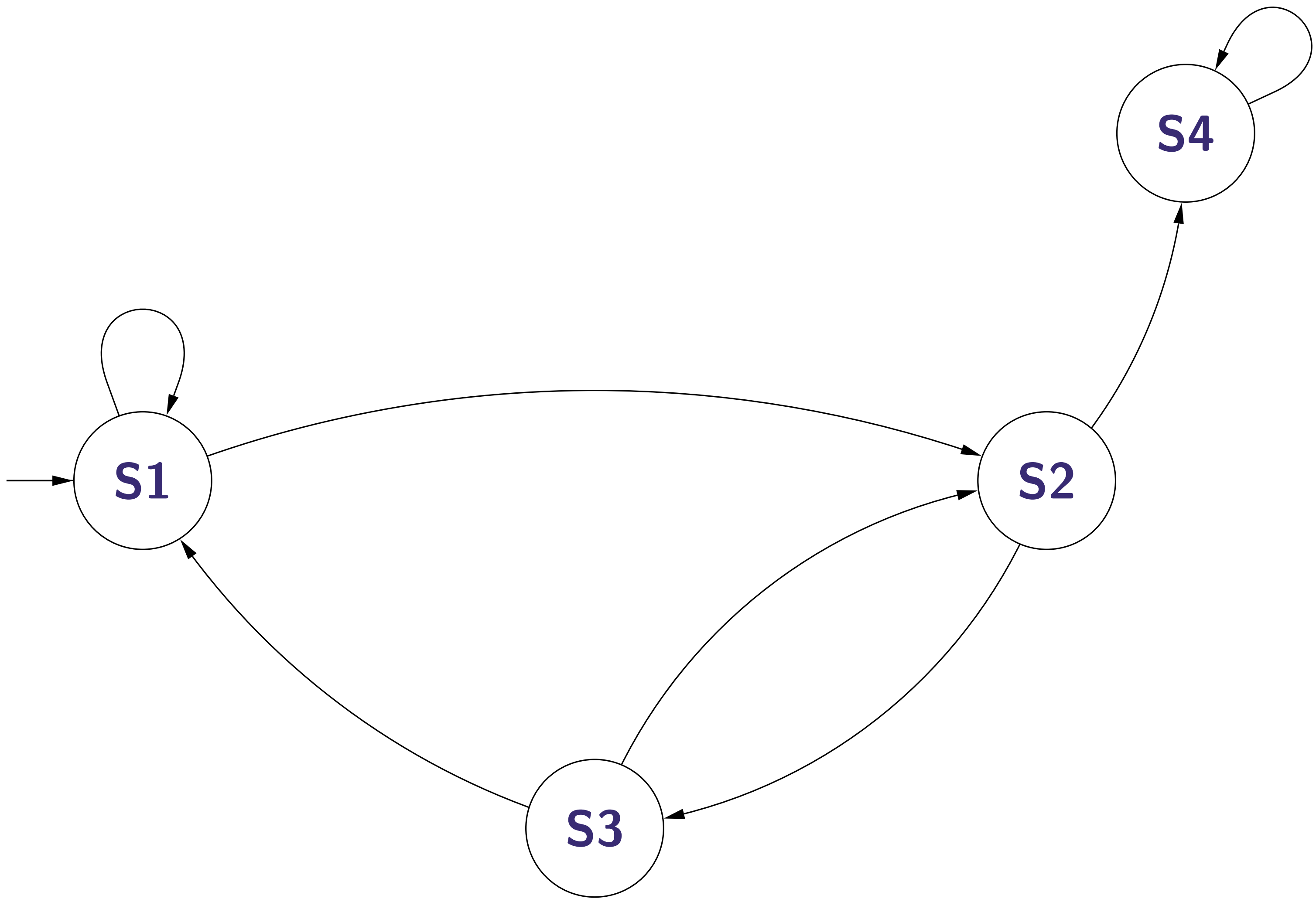
- Reachability for some specific classes of functions f.
- Abstractions of arbitrary systems using these specific functions.

Including some ongoing work:

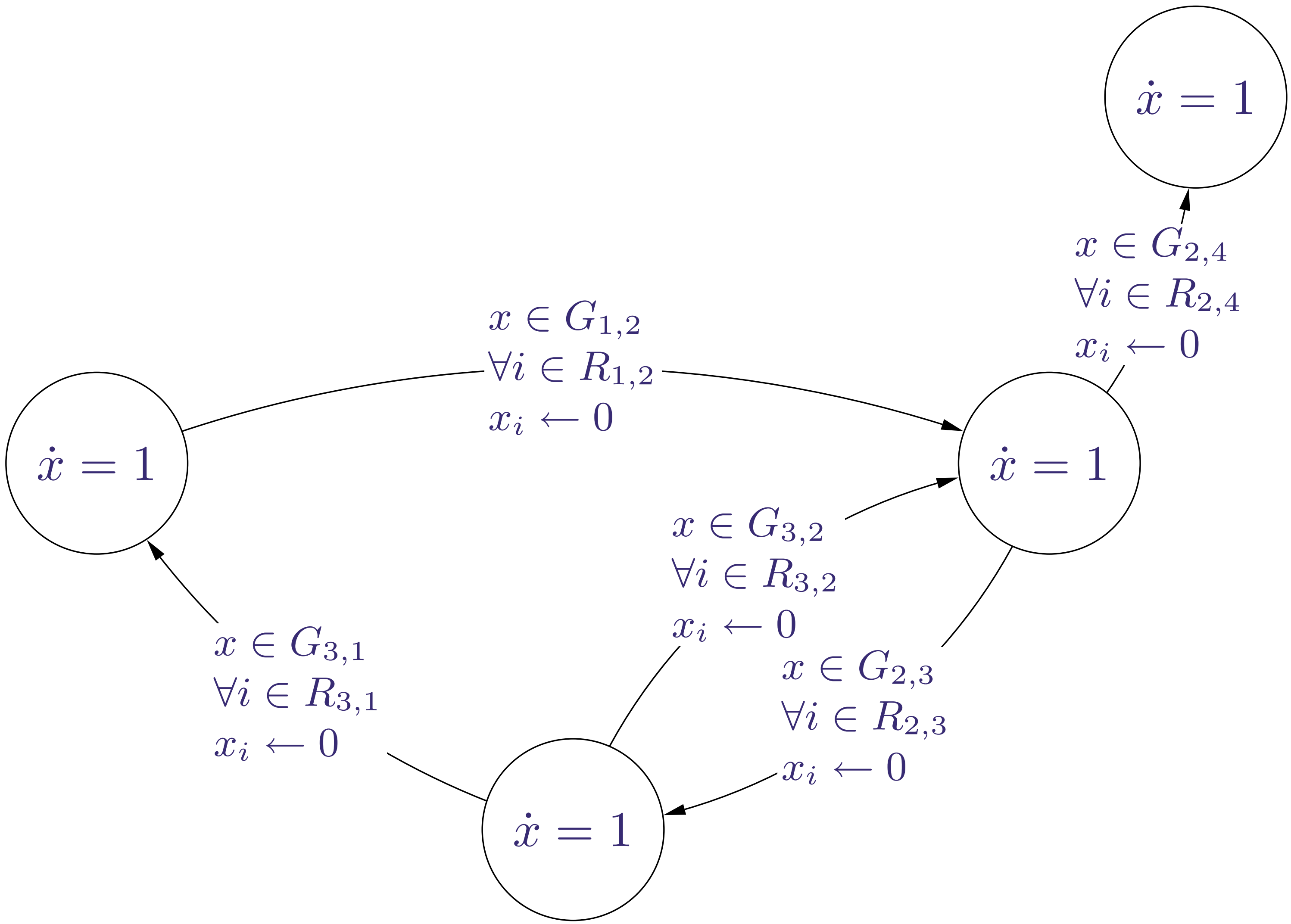
- On Linear Parameter Varying systems with Matthias Althoff and Bruce Krogh.
- On multi-affine systems with Radu Grosu, Flavio Fenton, James Glimm, Scott Smolka and Ezio Bartocci.

State of the Art

$f : \mathbb{R}^0 \longrightarrow \mathcal{P}\left(\mathbb{R}^0\right)$



$f(x) = \{1\}$


$\dot{x} = 1$
$x \in G_{2,4}$
$\forall i \in R_{2,4}$
$x_i \leftarrow 0$
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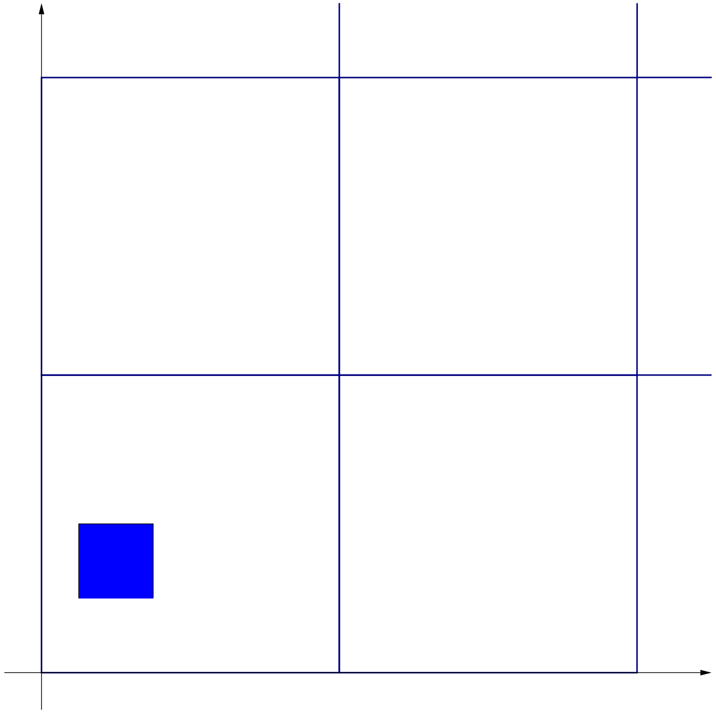
$\dot{x} = 1$

Linear Hybrid Automata

- simple continuous dynamics: conjunctions of linear constraints $a \cdot \dot{x} \leq b, \qquad a \in \mathbb{Z}^n, b \in \mathbb{Z}$
- All sets defined by Boolean combinations of linear constraints



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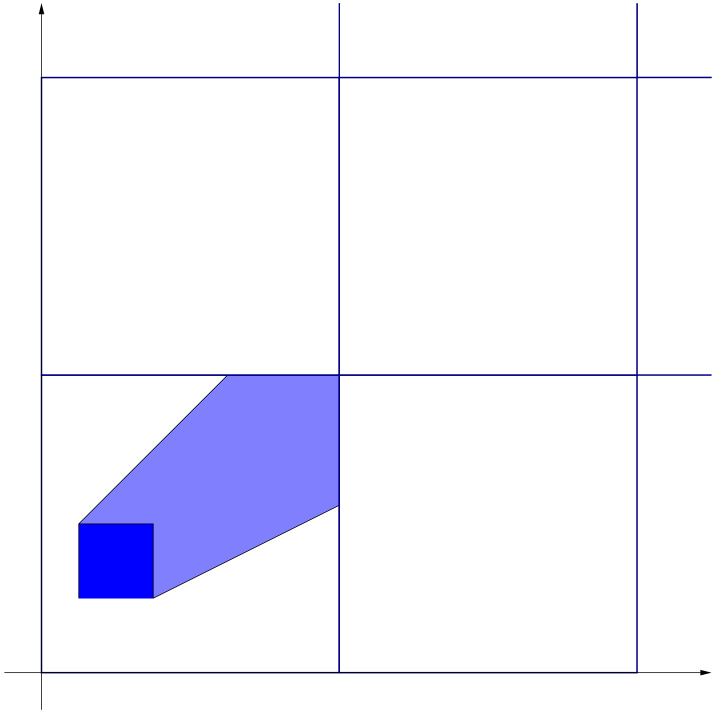

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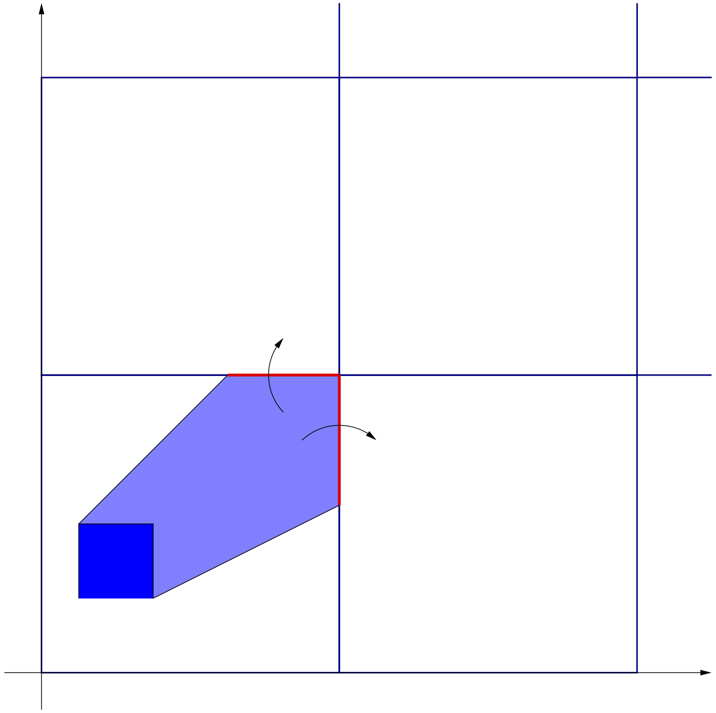
Post_d: discrete transition



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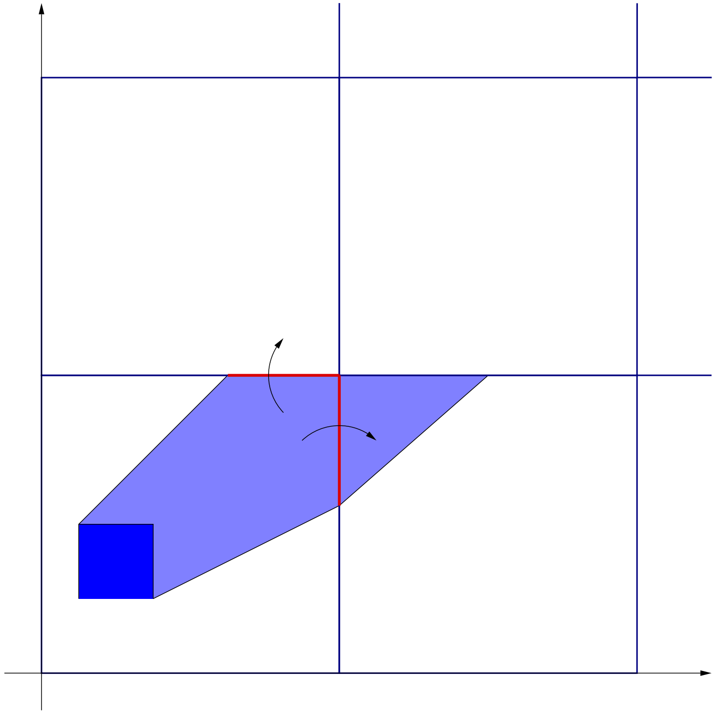

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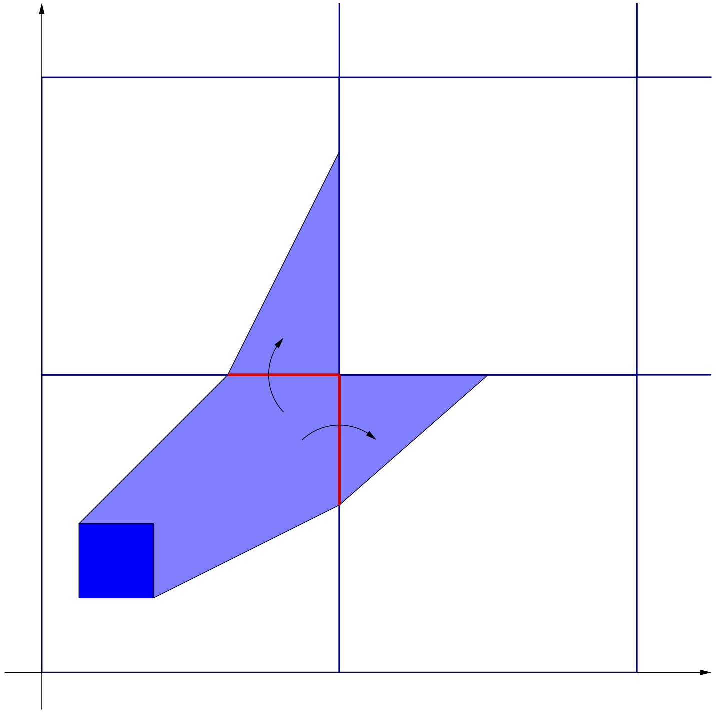

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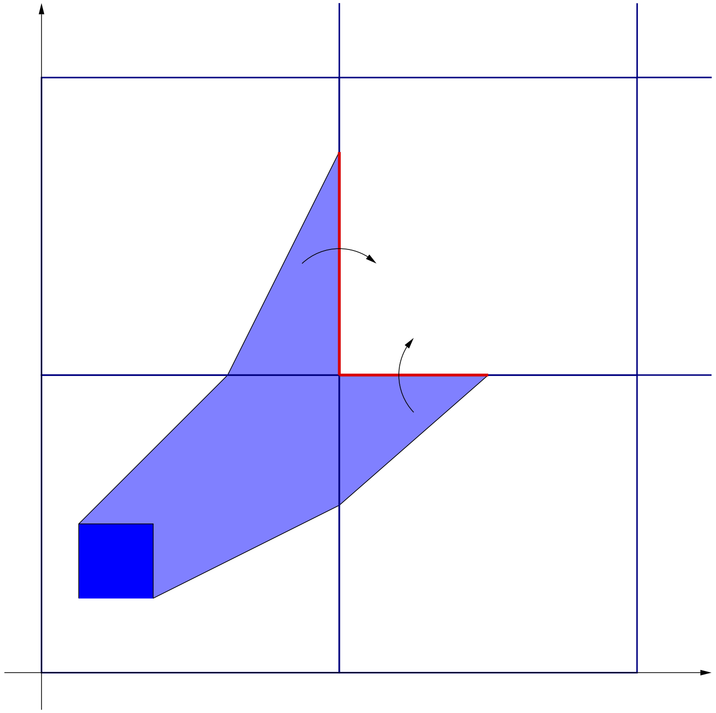
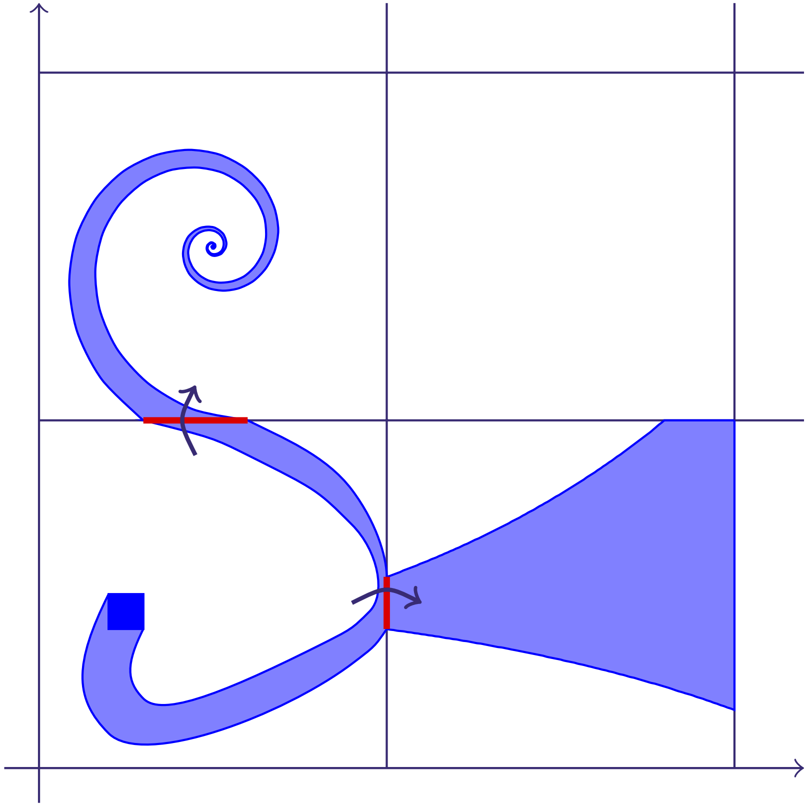


More expressive than LHA: $f(x) = 0\{x\} \oplus \mathcal{P}$

- Continuous dynamics: $\dot{x} \in A_q\{x\} \oplus \mathcal{U}_q$
- Switching hyperplanes or Polyhedral guards.



Reachability for LTI:

- Time discretization: $\dot{x} \in A\{x\} \oplus \mathcal{U} \longrightarrow x_{k+1} \in \Phi\{x_k\} \oplus \mathcal{V}$
- Computation of the N first terms of:

$$\Omega_{n+1} = \Phi\Omega_{n+1} \oplus \mathcal{V}$$

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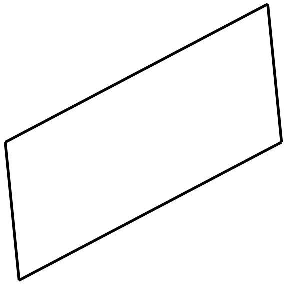

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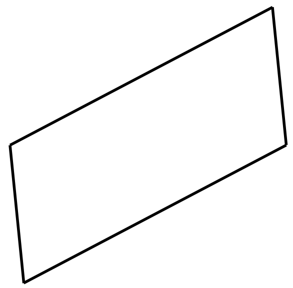

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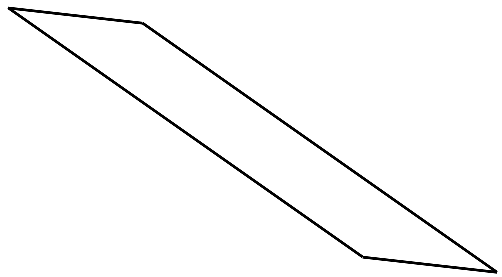

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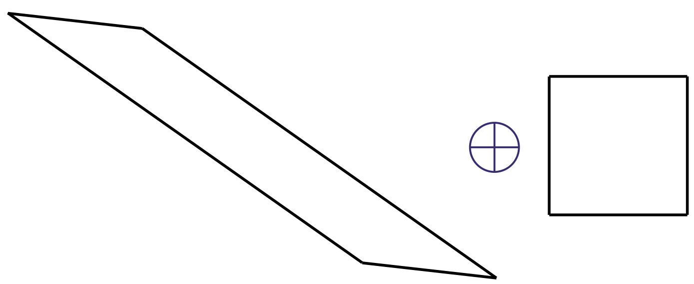

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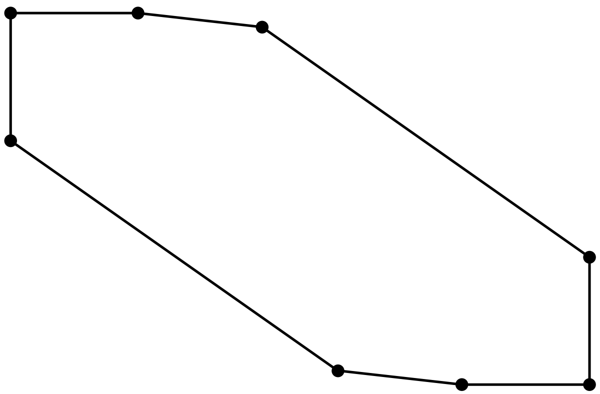

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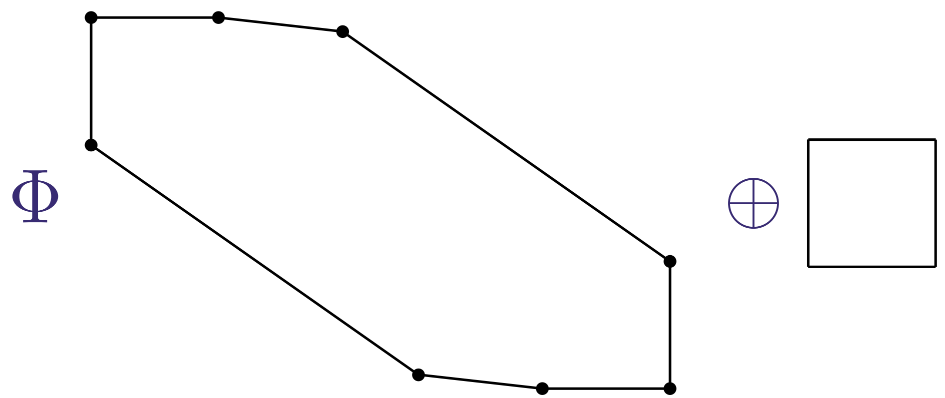

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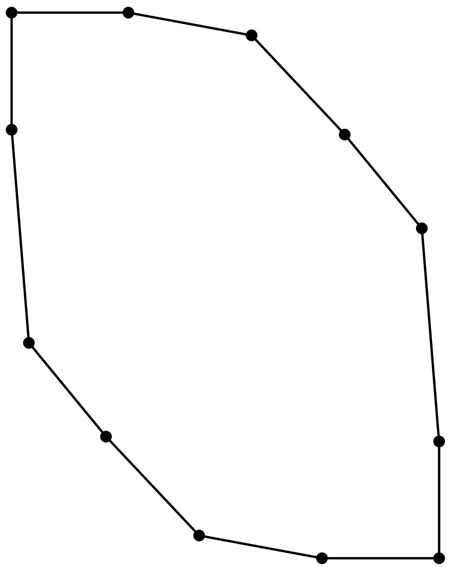

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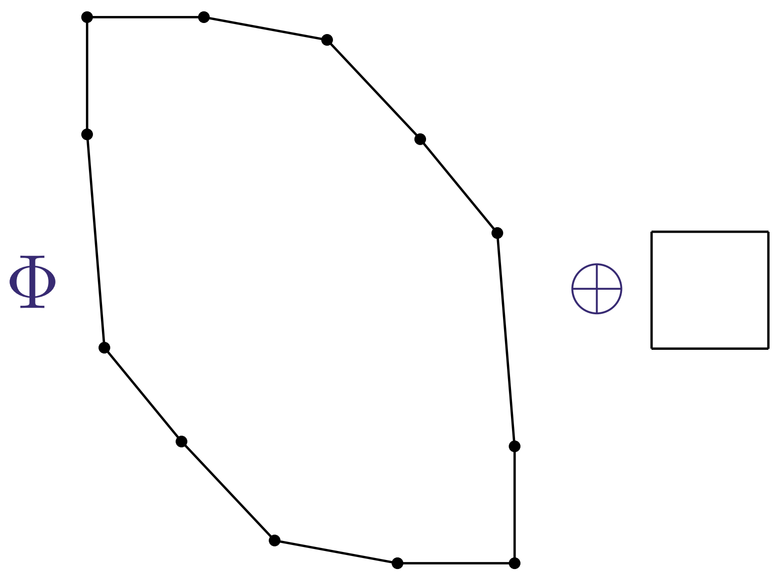

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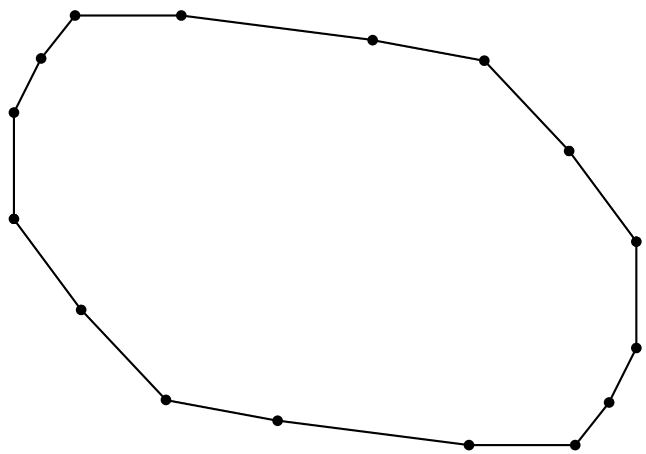

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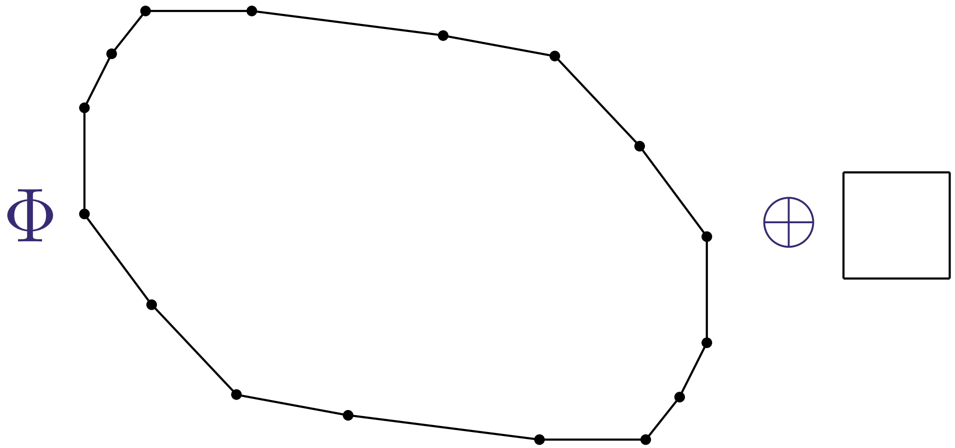

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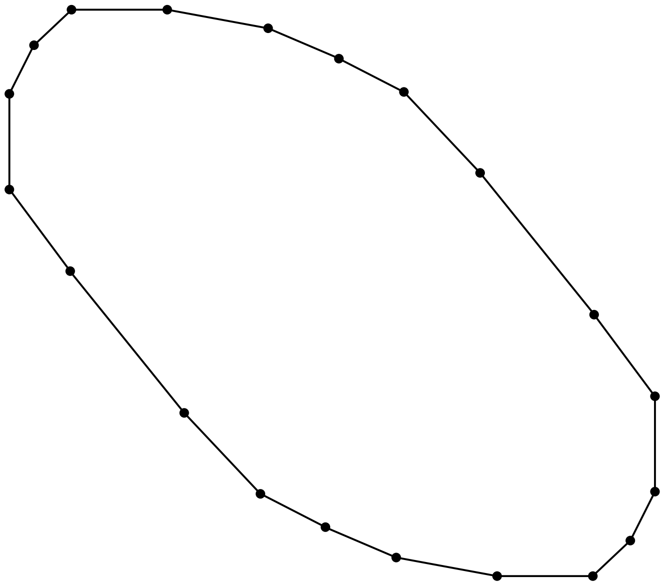

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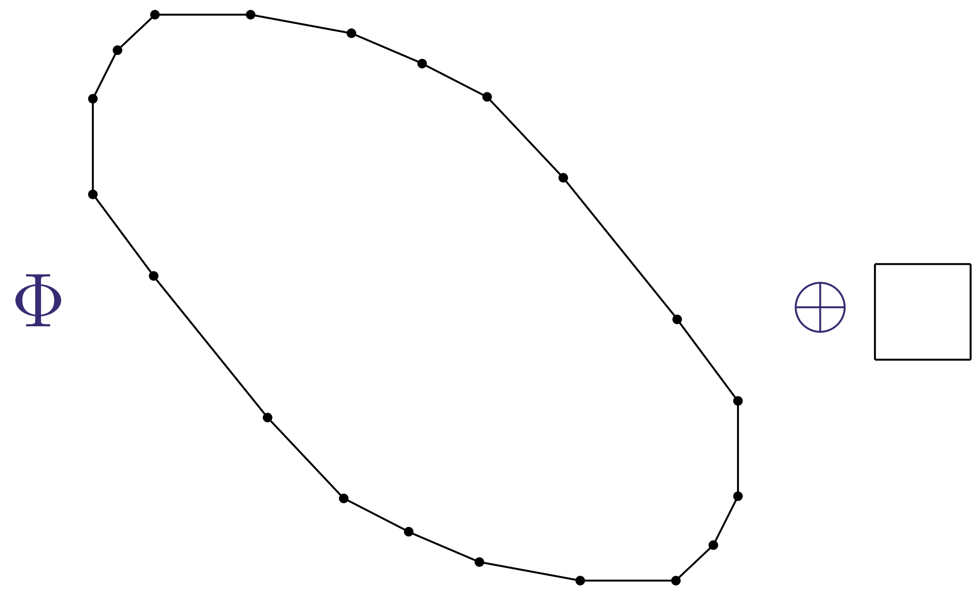

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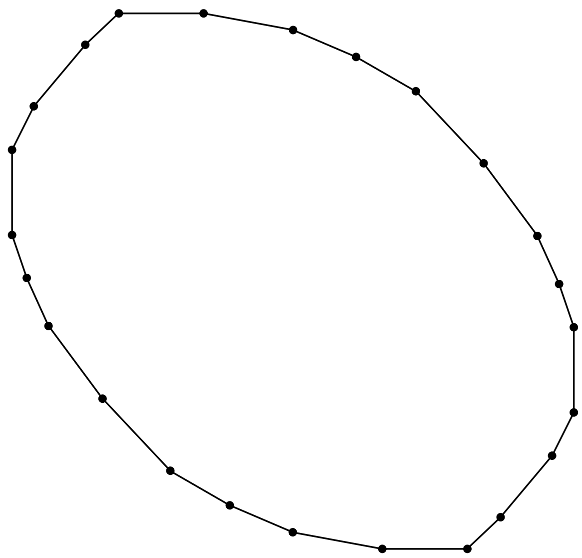

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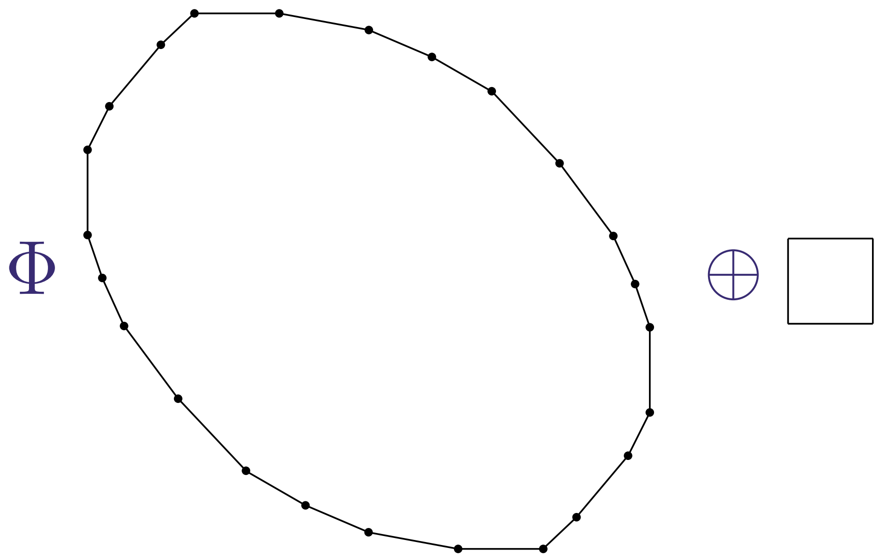

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Ω_{n-1} may have more than $\frac{(2n)^{d-1}}{\sqrt{d}}$ vertices.

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$f(x) = A\{x\} \oplus \mathcal{U}$

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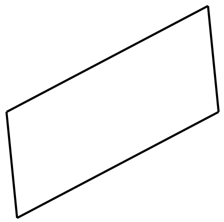

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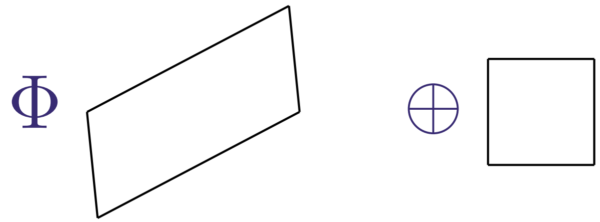

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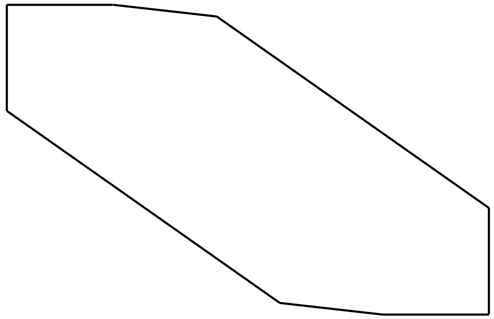

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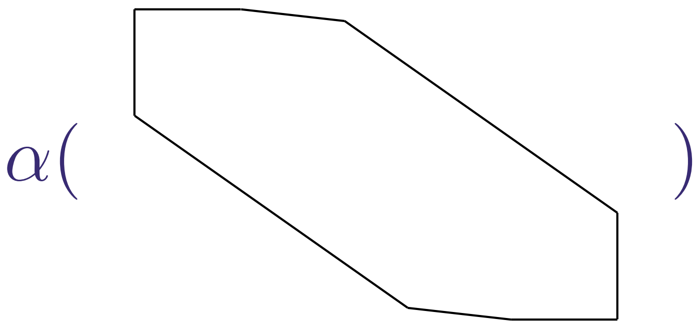

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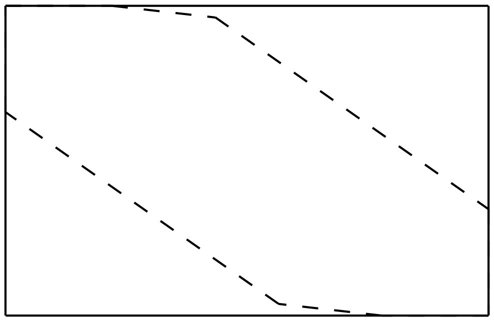

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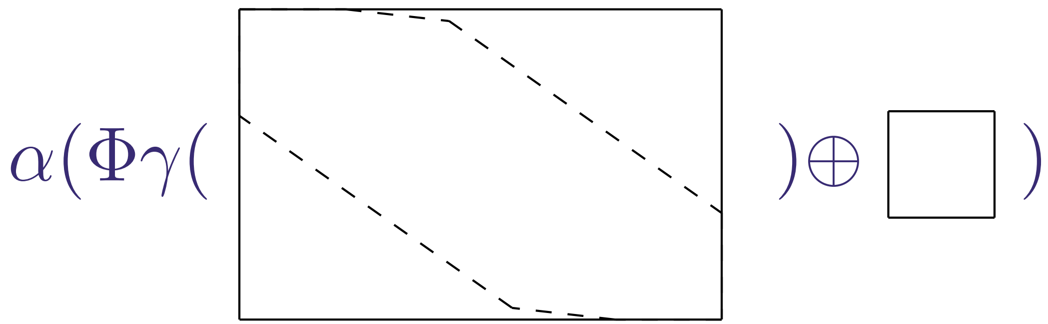

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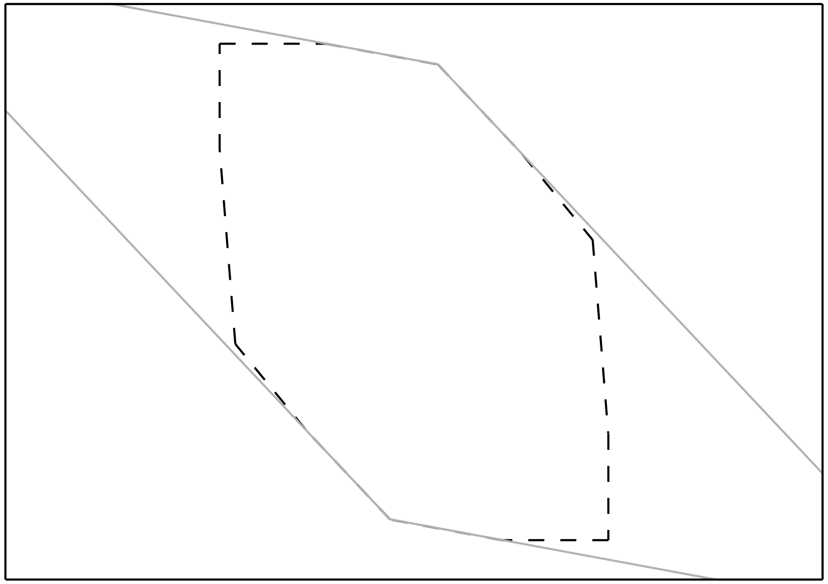

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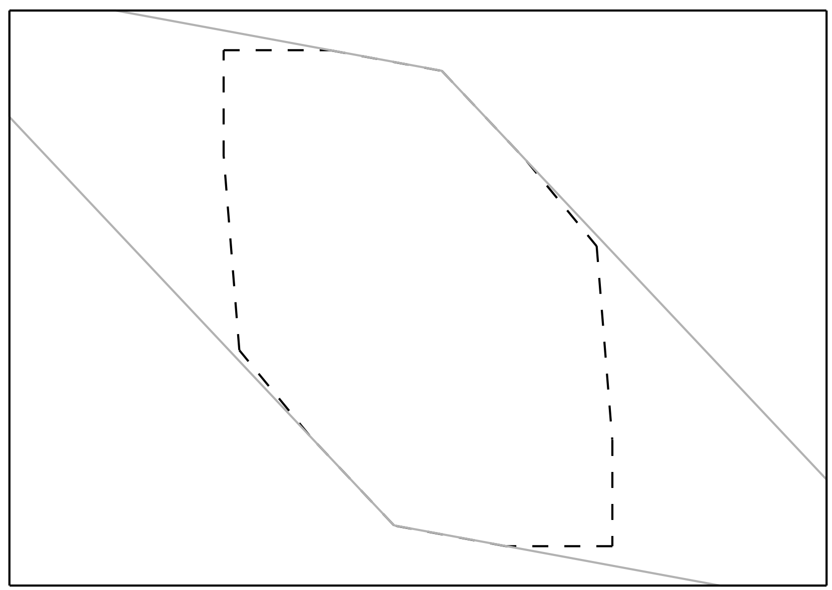
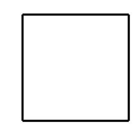
$\alpha(\Phi\gamma($  $) \oplus$  $)$

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- Time discretization: $\dot{x} \in A\{x\} \oplus \mathcal{U} \longrightarrow x_{k+1} \in \Phi\{x_k\} \oplus \mathcal{V}$
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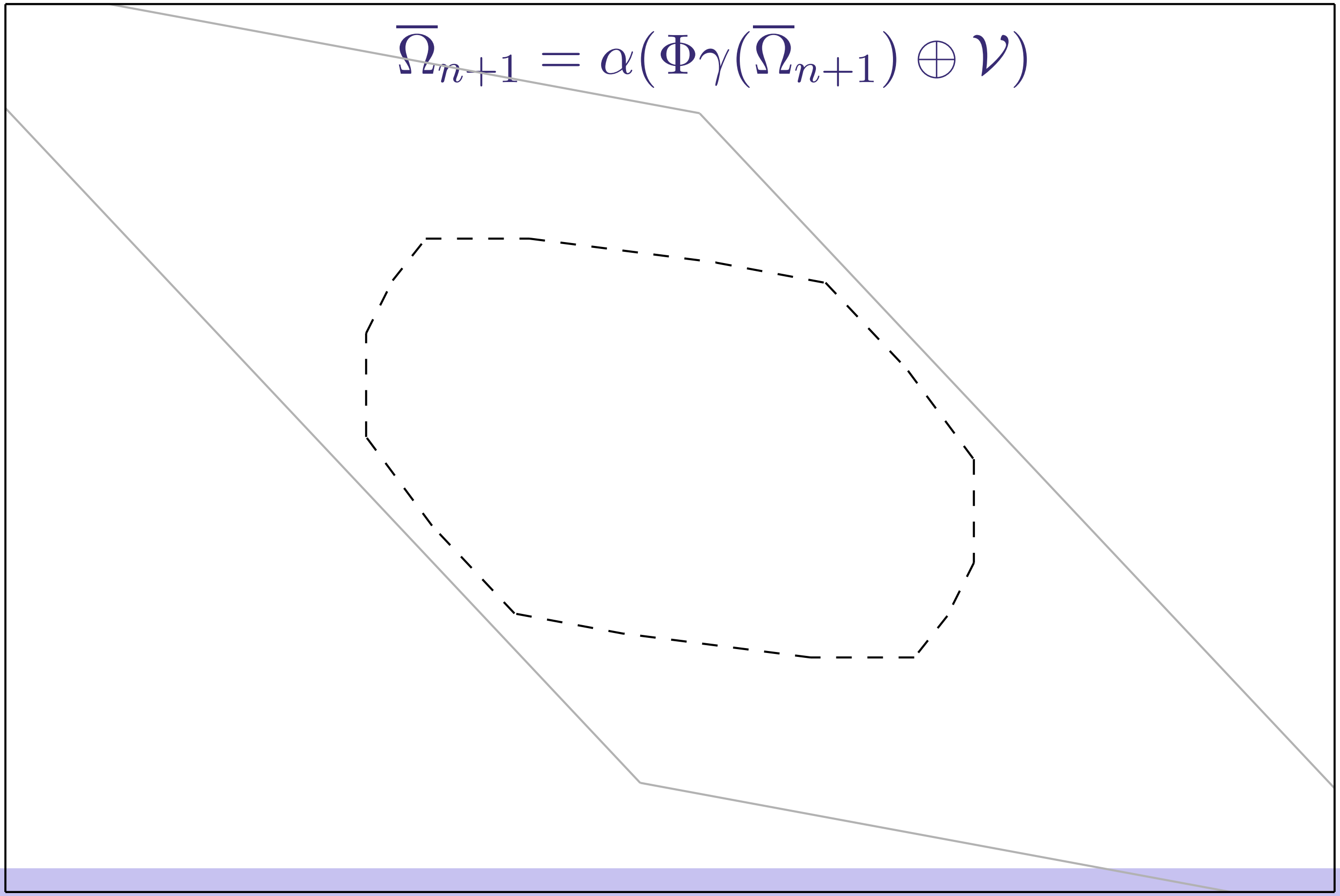

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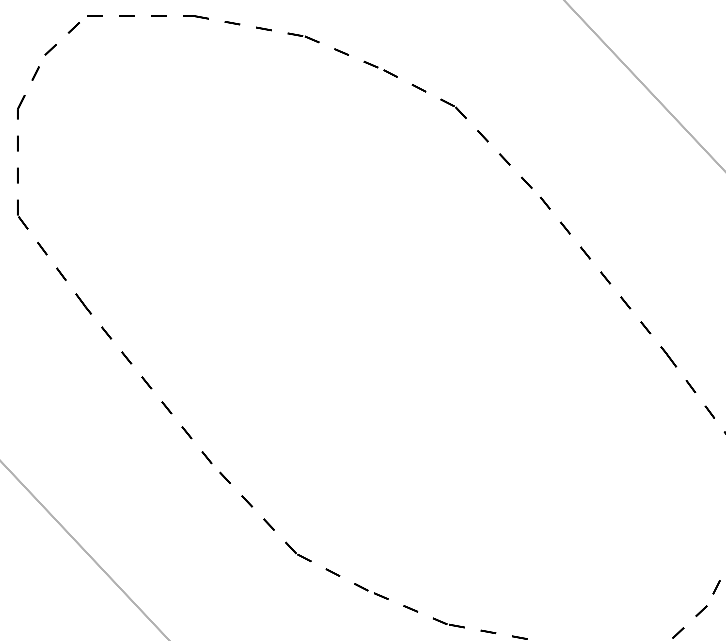

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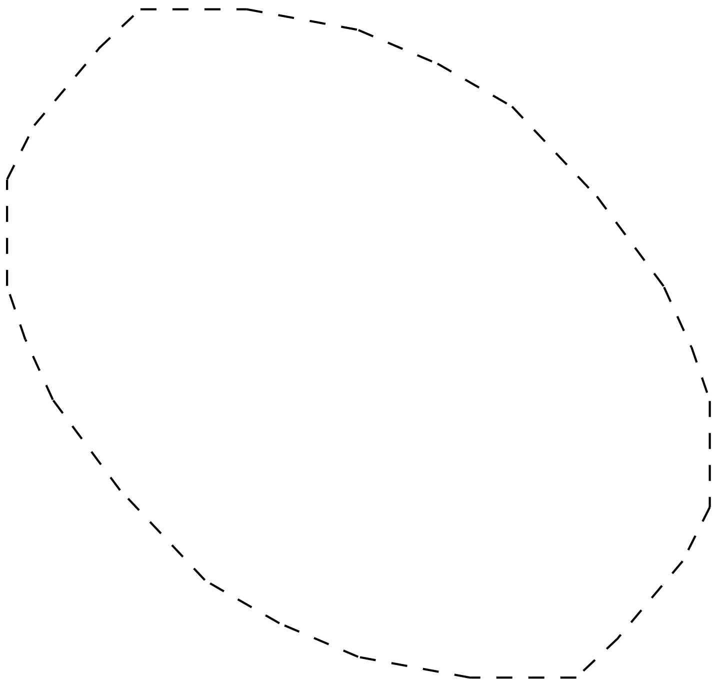

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$$T : \mathcal{X} \mapsto \Phi\mathcal{X} \oplus \mathcal{V}$$

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$$\mathcal{S}_0 = \{0\} \qquad \mathcal{S}_{n+1} = \mathcal{S}_n \oplus \mathcal{V}_n$$

Then: $\Omega_n = \mathcal{A}_n \oplus \mathcal{S}_n$

- $\mathcal{A}_i$ and $\mathcal{V}_i$ have a constant representation size.
- We can exploit redundancies of $\mathcal{S}_i$ (zonotopes, support functions).

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Approximations can still be interesting:

- We are only interested in one individual Ω_i.
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$$(\alpha \circ T \circ \gamma)^n = \alpha \circ T^n \circ \gamma$$

$$\alpha(\gamma(\alpha(\mathcal{Z})) \oplus \mathcal{Y}) = \alpha(\mathcal{Z} \oplus \mathcal{Y})$$

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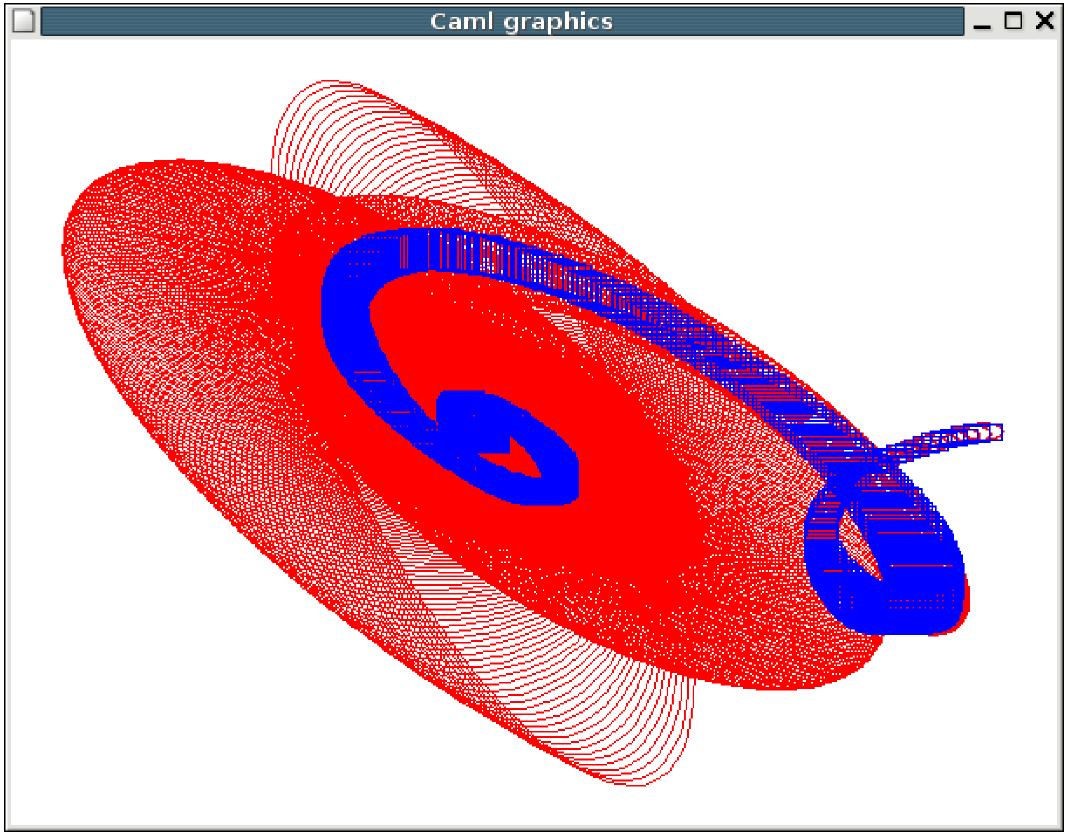

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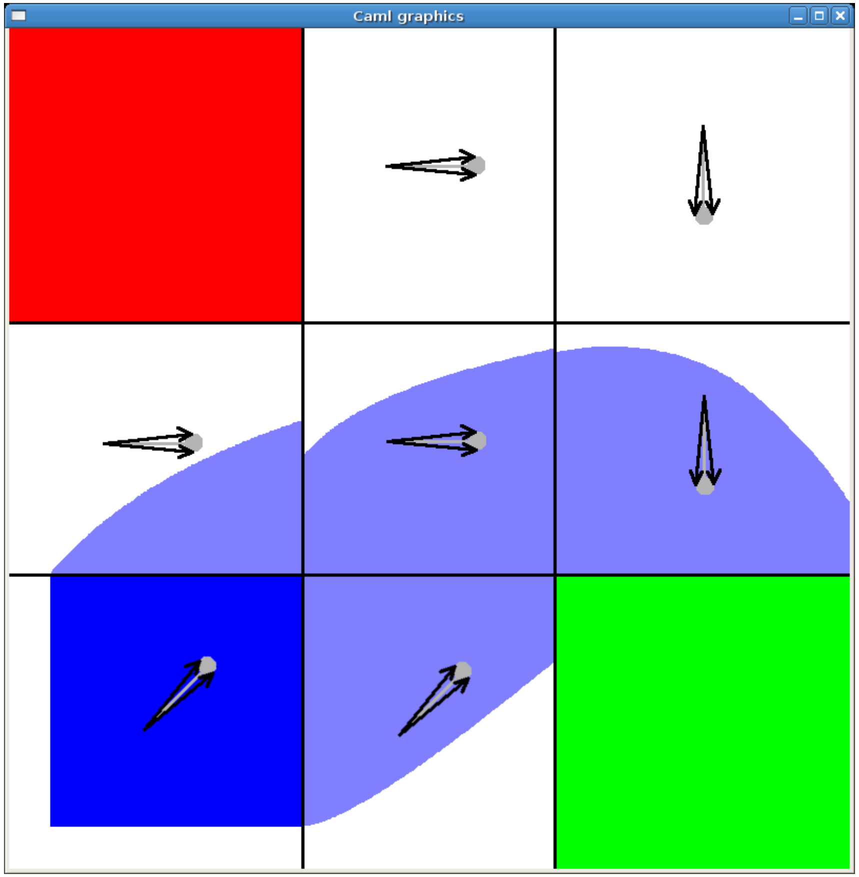

$f(x) = \{Ax \mid A \in \mathcal{A}\}$

More expressive than $f(x) = A\{x\} \oplus \mathcal{U}$ (in smaller dimension):

$$f(x) = \left\{ \begin{pmatrix} A & u \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix} \mid u \in \mathcal{U} \right\}$$

- Time discretization: $\dot{x} \in \mathcal{A}x \longrightarrow x_{n+1} \in \mathcal{M}x_n$
- Use of set representations in the space of Matrices.

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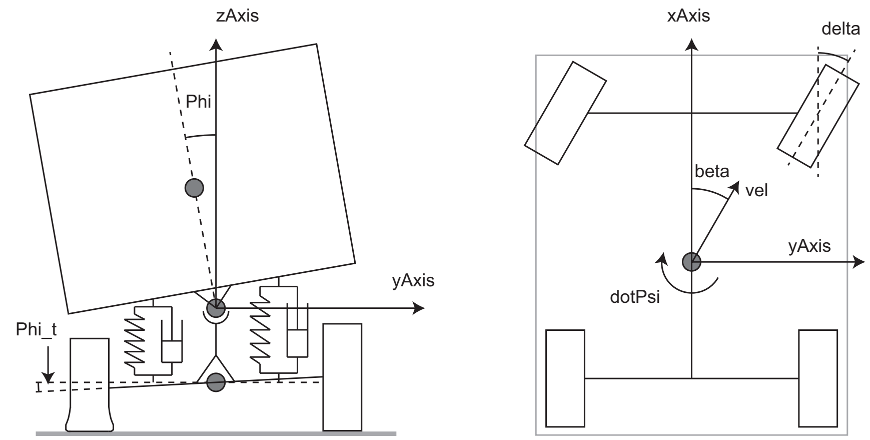

- 8 variables
- 3 discrete locations

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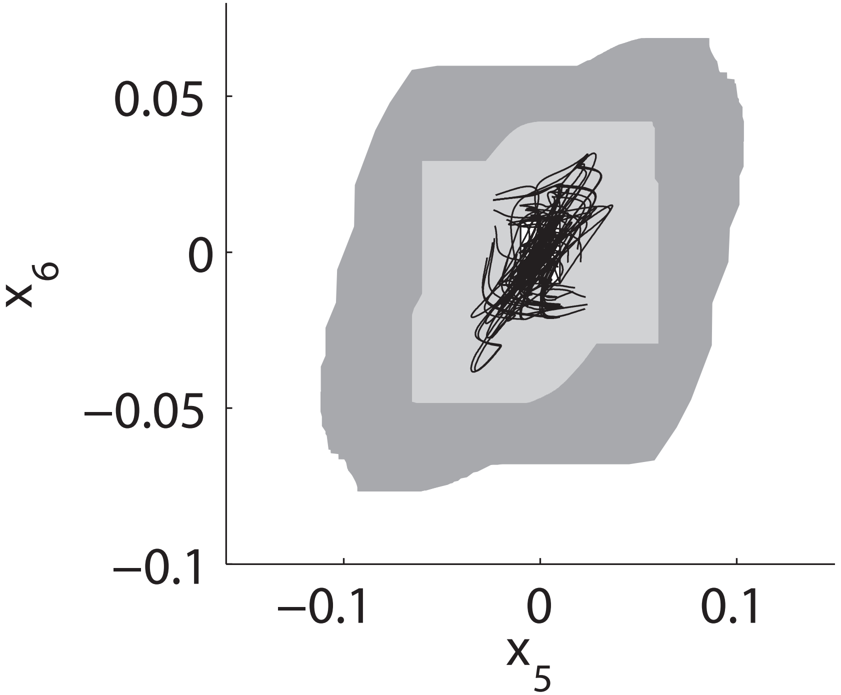

- 8 variables
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If we want to use similar techniques:

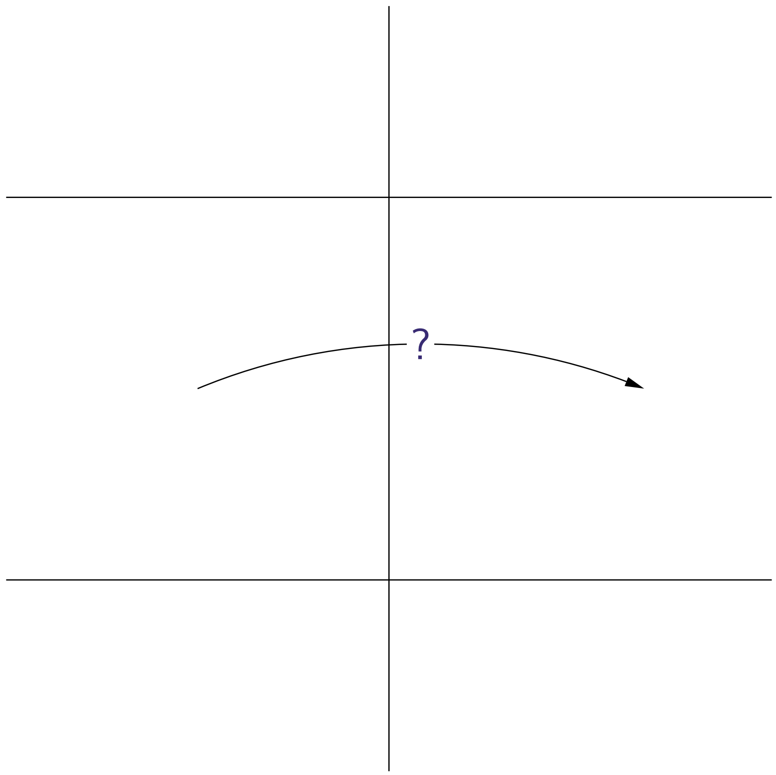
- Adapt integration schemes:

$$\mathcal{X} \mapsto \mathcal{X} \oplus \delta f(\mathcal{X}) \oplus \mathcal{E}$$

- Abstract

Abstraction

Rectangular partition.



We need to know if $f_i(\mathcal{G}) \cap \mathbb{R}^+$ is empty.

Smooth partition.

- Sign conditions on a set of functions and their derivatives.
- No transition from $(x > 0, \dot{x} > 0)$ to $(x < 0, \dot{x} > 0)$

We need to check emptiness of the cells.

Timed automata.

- Partition of the state space in slices
- Clocks measure time to get from one slice to the other
- We need to know upper and lower bounds for $f_i(\mathcal{S})$.
- Easier when Lyapunov functions are availables

LHA

- Polyhedral partition
- For each cell $\mathcal{C}$ of the partition, we need to know $f(\mathcal{C})$

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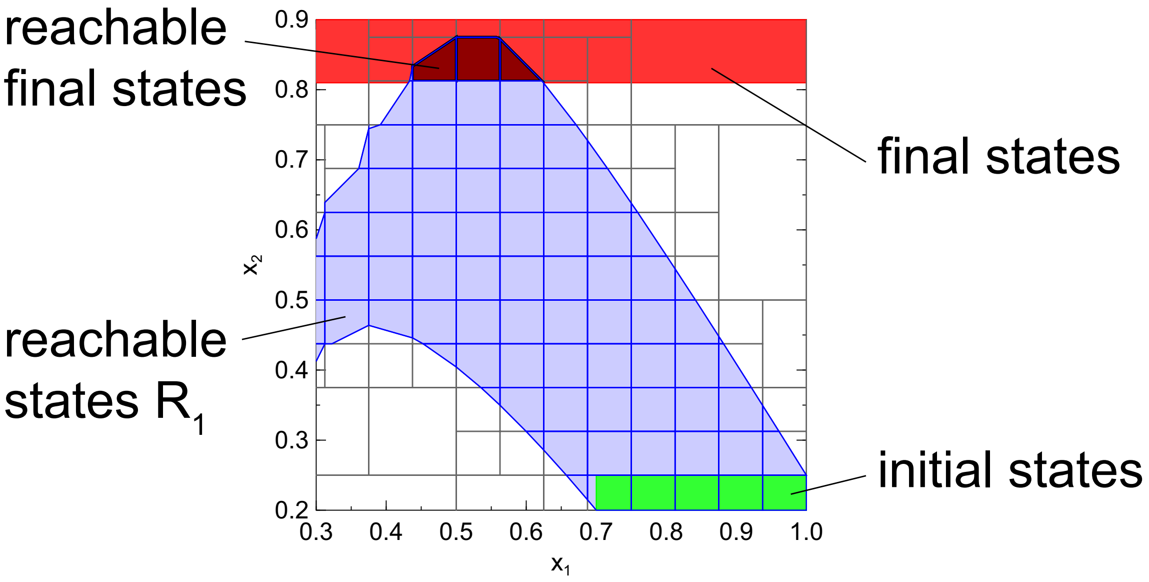

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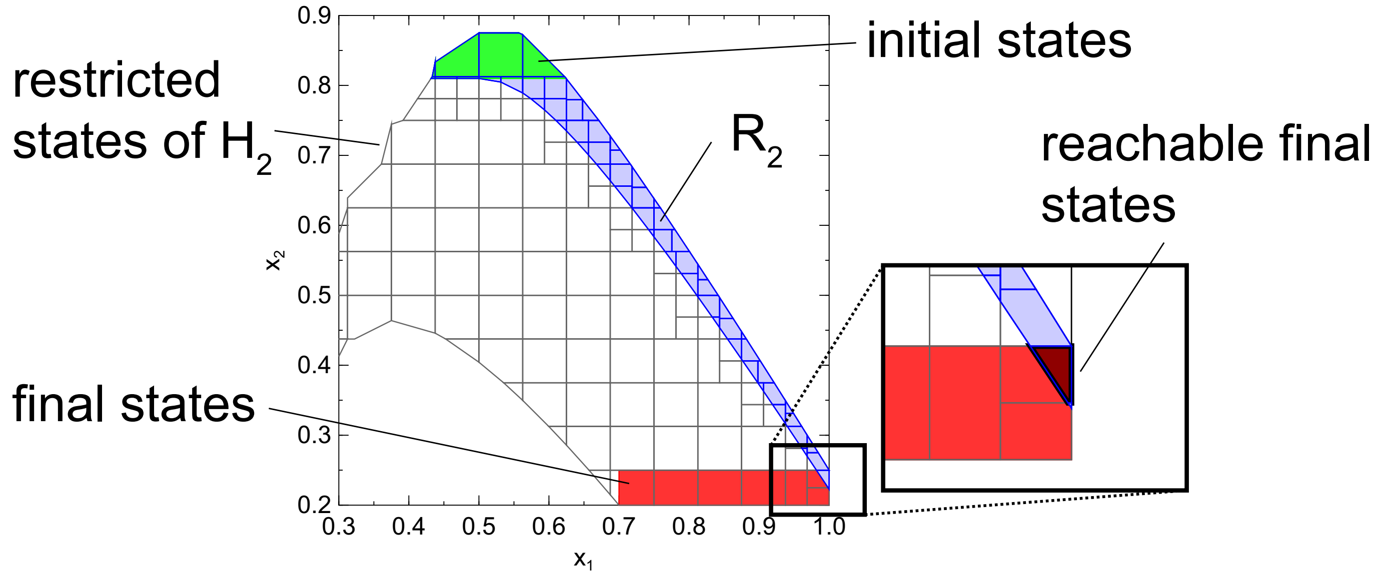

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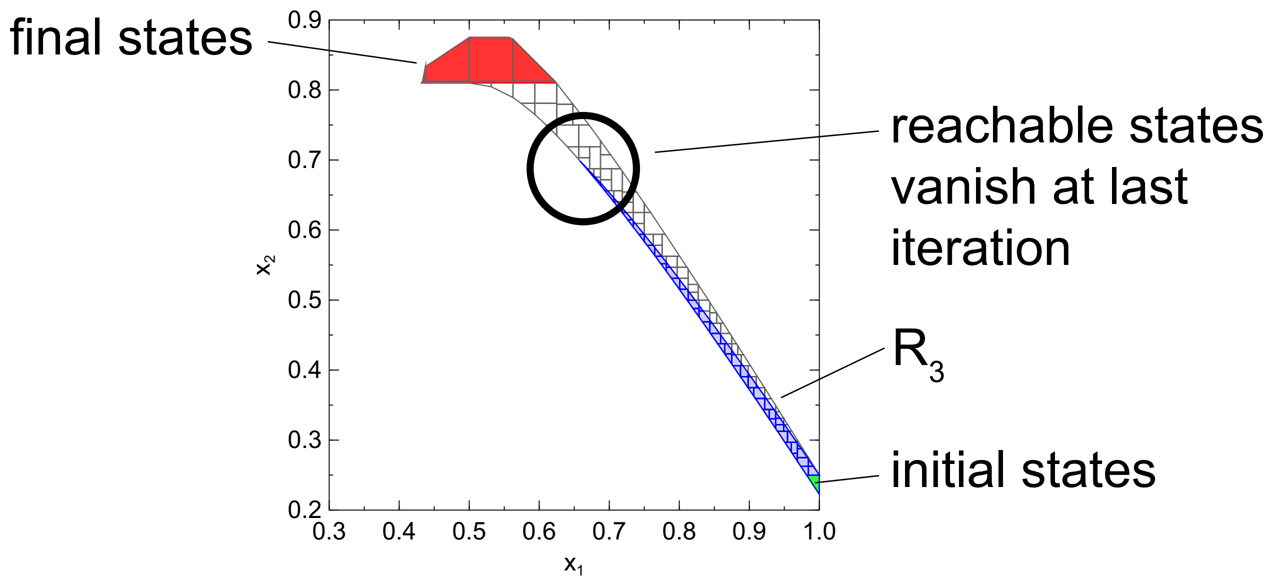

$\bar{f}(x) = A\{x\} \oplus \mathcal{U}$

For each cell $\mathcal{C}$ of the partition:

- Choose linearization A
- Compute $\mathcal{U} = \{y - Ax \mid x \in \mathcal{C},\, y \in f(x)\}$

We want $\mathcal{U}$ to be as small as possible, how do we choose A?

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NYU

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If we find a subset of variables such that:

- f is linear in these variables
- no product of two of these variables appear in f

We do not need to partition along these variables.

Conclusion

Choosing the right abstraction is rarely easy.

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- modifying the number of continuous variables
- combining different classes of abstractions

Thank you